Monogamy, Polygamy, and the Regularity of Emergent-Geometry Graphs

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A study of mutual-information metrics under open dynamics

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Abstract

A recurring construction in emergent-geometry programs weights the edges of a graph by the pairwise mutual information of qubits at its vertices and reads distance off the result. Three independent programs of this kind impose the regularity that a manifold limit needs rather than deriving it: one by restricting to a hand-picked class of states, one by a hand-built energy penalty, one by axiom. This study asks whether the regularity can be derived instead, and answers in layers. Kinematically, no: total mutual information is polygamous, a single classical bit read by N parties gives every pairwise edge full weight at fixed local entropy, so no per-vertex budget caps the degree of an I-weighted graph, and the same construction cannot even distinguish a separable broadcast state from a GHZ state. Re-key the weight to a monogamous measure and half the assumption becomes a theorem: squashed entanglement or tangle gives deg <= ln 2 / eps, uniform in system size and state. Whether dynamics can then sustain a connected monogamous graph is answered by a state-level automaton study across ten pre-registered rounds. The findings, each forced by a falsification: dilution channels worth the name are unital, so a source-free automaton dies at the entropy ceiling and genesis functions as the negentropy supply. Pair-replacement genesis cannot weave, its rate cancels. Fixed-rate genesis loses to production that scales with N. Volumetric genesis yields a live phase at a derived fixed point r* = lambda / (lambda + b). And in that live phase the monogamous degree is pinned near one across every dial, source, rate, dilution, and coupling strength, with its maximum exactly at the half-maximum set-point coupling, while the polygamous graph spreads with exponential hop decay and the monogamous weight stays confined to direct partners. A closed-form retention curve R(theta) for a Bell edge under one foreign firing derives three coupling phases, including complete annihilation under the full gate. The conclusion is an elimination: within two-body sequential dynamics a percolating monogamous weave does not exist in the mapped space, and the one untested lever is intrinsically multipartite interaction. Every negative result is reported with its mechanism. The anomaly ledger of the campaign, fifteen entries, all reproduced, is part of the record.

Status conventions. Every claim in this study carries one of five words. *Proven*: a theorem with a written proof. *Derived*: a first-principles argument with stated assumptions, checked against data. *Mean-field*: a derived law whose closure step is a factorization assumption. *Observed*: a reproduced measurement. *Open*: named and unsolved.

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1. The setup and the problem

1.1 The model class

Take a countable vertex set, one qubit per vertex, and a global state rho. For any pair of vertices define the mutual information

$$I(i:j) = S(\rho_i) + S(\rho_j) - S(\rho_{ij}),$$

with S the von Neumann entropy in nats. An edge exists where the weight clears a floor, $W(i:j) \geq \epsilon$, and distance is read from edge lengths $\ell = -\ln(W / W_{max})$ by geodesic closure. The construction is not a straw man. Cao, Carroll, and Michalakis build spatial geometry from exactly this weight. Quantum graphity’s dynamical graphs carry the same ambition with an energy functional in place of a state. And the companion framework to this study takes the construction as its Definition 3 (Halldén 2026a). All three need the graph to be regular, bounded degree with decaying weight, before any manifold can condense out of it.

1.2 The problem

None of the three derives the regularity. Cao, Carroll, and Michalakis restrict by hand to a class they name redundancy-constrained states. Quantum graphity’s low-energy local phase is stabilized by a degree penalty written into the Hamiltonian. The companion framework assumes regularity as an axiom, A4, with the cost stated openly, and its own simulations show the bare dynamics densifying toward a near-complete graph, in direct tension with the axiom. Three programs, three hands on the same scale.

The question of this study: is the hand necessary? Concretely, does any budget cap the degree of the I-weighted graph, and if a budget exists for some other weight, can a physical dynamics sustain a connected graph under it?

1.3 What the answer will look like

The kinematic half is settled in Section 2 by two short theorems: no budget for I, a sharp budget for monogamous weights. The dynamical half occupies the rest of the study and does not go the way the theorems suggest it might. The sustained finding, forced round by round against the author’s own pre-registered expectations, is that two-body sequential dynamics pins the monogamous degree near one no matter which dial is turned. The mechanisms behind the pin are the study’s main content.

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2. Kinematics: two theorems and a diagnosis

2.1 Theorem 1 (no budget for mutual information). Proven.

Claim. For every N there is a state of N + 1 qubits with $S(\rho_0) = \ln 2$ fixed while the pairwise sum $\sum_j I(0:j) = N \ln 2$ grows without bound. Consequently the bounded degree of an I-weighted graph is not derivable from any per-vertex information budget.

Proof. The classical broadcast state,

$$\rho = (1/2)\, |0\ldots0\rangle\langle0\ldots0| + (1/2)\, |1\ldots1\rangle\langle1\ldots1|,$$

has every single-qubit marginal maximally mixed and every pair marginal equal to the two-bit correlated mixture, so $I(0:j) = \ln 2$ for every j simultaneously. All N edges clear any floor $\epsilon \leq \ln 2$ at once. The state is a mixture of products, hence separable. Every pairwise entanglement measure vanishes on it. QED.

Numerics, from the archived cross-check at N = 5: each pairwise I = 0.693147, sum 2.772589, pairwise negativity 0.000000.

Two sharpenings fall out of the same family. First, the unbounded part is pure redundancy and invisible to the collective edge: Araki-Lieb gives $I(0 : \text{rest}) \leq 2 S(\rho_0) \leq 2 \ln 2$ always, and the broadcast state realizes $I(0 : \text{rest}) = \ln 2$ against a pairwise sum four times larger at N = 5. One record, read by everyone, is counted once by the collective mutual information and N times by the edges. The degree of the I-graph counts readers, and records have unlimited readers. Second, coherence changes nothing at the pairwise level: the pure GHZ state on the same qubits has identical pair marginals, I = ln 2 and zero entanglement per pair, with its entire budget on the collective edge. Pairwise I cannot tell a separable broadcast from a globally entangled GHZ. The metric's weight is blind to where correlation lives.

Corollary. Under W = I, the degree-N configuration is kinematically free of charge. The densification that the companion framework's own automaton exhibits is thereby explained rather than anomalous, and the hand each of the three programs applies is explained with it: an I-weighted construction has nothing internal with which to refuse the broadcast.

2.2 Theorem 2 (the budget lemma). Proven.

Claim. Key edge persistence to squashed entanglement, E_sq(i:j) >= eps. Then for every global state and every vertex hosting a qubit,

deg(i) <= E_sq(i : V i) / eps <= S(rho_i) / eps <= ln 2 / eps,

uniform in |V|. On qubits the tangle gives the same conclusion with budget 1.

Proof. Christandl and Winter's monogamy, E_sq(A : BC) >= E_sq(A:B) + E_sq(A:C), iterates over a partition of V i into singletons. The trivial extension in the squashed infimum gives E_sq(A:B) <= I(A:B) / 2, and Araki-Lieb bounds I(A:B) by 2 min(S_A, S_B), so E_sq(i : V i) <= S(rho_i) <= ln 2. Divide the sum by the floor. The tangle variant substitutes the Coffman-Kundu-Wootters inequality in Osborne and Verstraete's n-qubit form, Sum_j C^2(i:j) <= 1. QED.

Raw entanglement of formation is not monogamous in this form and is excluded. The theorem stands on the two airtight choices. One consequence stands alone: the budget forces per-edge weight below ln 2 / deg, so a near-maximal edge drives its vertex toward degree one. Strong entanglement monopolizes. This sentence, kinematic here, returns as the engine of the dynamical results.

2.3 The dichotomy

An I-weighted graph admits no budget and densifies for free. A monogamously weighted graph has bounded degree as a theorem, at the price of one definitional change to the metric's weight. The choice is forced in one direction only if the dynamics can sustain monogamous weight on a connected graph, and that single question is what the remaining sections decide. The verdict, stated now so the reader can watch it arrive: sustained, yes. Connected, no, and the reasons are sharper than the hope was.

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3. Dynamics I: fuel, source, and the live phase

3.1 The automaton, at state level

The dynamics under test is minimal and fully specified. Per step: vertices are paired by a random matching weighted toward short geodesic distance. Each matched pair receives a two-qubit unitary built from a controlled rotation dressed with a Hadamard and a T gate, so the dynamics is universal and injects non-Clifford resource. Every qubit then passes through a single-qubit dilution channel of strength p. A source acts last. Measurement reads all pairwise marginals. The coupling strength of the unitary is the subject of Section 4 and is the study's most consequential dial. Simulations run exact density matrices to twelve qubits and pure-state trajectories, an exact unraveling of the dilution channel, to sixteen, with ensemble marginals recovered by averaging. Ten rounds were pre-registered before their builds. Every number below was recomputed from archived outputs before being quoted here.

3.2 The unitality theorem, and what genesis is. Derived, machine-checked premise.

Every channel that deserves the name dilution is unital: depolarizing, reset toward the maximally mixed state, and pure dephasing all satisfy $\Lambda(I) = I$, checked to machine precision, and reset at strength $4p/3$ turns out to be depolarizing at strength p exactly, the same map in two parametrizations, a caution for anyone counting channel variation as robustness. Unital channels never decrease von Neumann entropy. Unitaries preserve it. A source-free automaton therefore has monotone global entropy, and the observed endpoint is the ceiling $N \ln 2$, where every unitary acts trivially and the dynamics is inert. The numbers: entropy 4.073 of a 4.159 ceiling by step fourteen at six qubits under depolarizing, 4.158 under dephasing, with the strongest edge decaying from 0.90 to 0.01 while pairs fire throughout (Figure 1). Dephasing dies faster, not slower, because the Hadamard and T rotate every protected diagonal sector into coherence each step, and the composite acts as an effective depolarizer. The universal gate set that gravity-like response demands is precisely what forecloses the classical escape route.

Two consequences. Purity is the fuel of correlation, and dilution only spends it. And a scalar update rule that raises an edge weight toward a set-point conceals a source, because no unitary can raise correlation at the ceiling. Genesis, a fresh balanced pure pair, is the negentropy supply, load-bearing for the live phase rather than a historical trigger.

3.3 The genesis chain: three source models, two exclusions, one law

Replacement cannot weave. Mean-field, measured on the bound. Replace a random pair per step with a fresh Bell state and the fuel gauge stabilizes at roughly half the ceiling, alive, while the graph stays empty. The reason cancels cleanly: each replacement adds one edge and erases the adjacency of two vertices, so gain and loss share the rate and the rate drops out, steady mean degree

$$d = (n_{inj} + m) / (2\, n_{inj} + \text{decay term}) \to 1/2$$

as the rate grows. Measured degrees 0.43 to 0.55 across a fourfold rate sweep sit on the bound.

Fixed-rate growth loses to N. Derived, crossing observed. Let the graph grow instead: fresh pure pairs at g births per step supply $2 \ln 2\, g$ nats of negentropy while dilution produces on the order of $0.1 N$ nats per step. The balance point N^* is about fourteen g, and the growth trajectories cross it where the arithmetic says: pure-pair births hold structure at six qubits and lose it by twelve, while copy-births, which add capacity but no purity, never hold anything at all. A per-step source of order one is a candle against a fire that scales with the house.

Volumetric genesis and the r* law. Derived with a bracketed coefficient. Ratio equation exact. Make the source volumetric, $b N / 2$ pairs per step, and both sides scale together. The production coefficient follows from operator weight: depolarizing contracts a weight-w Pauli string's purity contribution at rate $8 p w / 3$, the dynamics writes fresh structure at weight two and scrambles it slowly, so the deficit relaxes as $dS = \lambda (S_{max} - S)$ with λ between $16 p / 3$ and $8 p$. With capacity growing at b per unit, the fuel ratio $r = S / S_{max}$ obeys, exactly given the production law,

$$dr = \lambda (1 - r) - b r, \quad r^* = \lambda / (\lambda + b).$$

Fitted values from two dilution strengths give λ / p of 6.0 to 6.2, inside the bracket. Measured trajectories approach the predicted bands from below (Figure 2). The live extended phase exists at every b, with quality set by a dial rather than a threshold. One decoupling completes the picture: raising b buys fuel, and only fuel. The graph's density saturates against the source rate, because the weave is woven and unwoven by the interaction dynamics at its own pace. What that pace is, and why it pins, is the next two sections.

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4. Dynamics II: the coupling, three phases, and the set-point

4.1 The exact retention curve. Exact, machine-verified for the model gate.

Everything about the weave turns on one exactly computed object: the tangle retained by a Bell pair when one of its legs fires with a stranger at coupling theta,

theta / theta*: 0.25 0.50 1.00 2.00 full R(theta): 0.972 0.890 0.608 0.047 0.000

computed exactly on three qubits, where theta* is the coupling calibrated so that one firing on a fresh pair lands at mutual information ln 2, half the pair maximum (Figure 3, upper panel). The last entry carries the campaign's earlier rounds in one number: the full-strength gate does not dilute a prior edge, it deletes it in a single foreign firing. Six earlier rounds of this study ran full-strength gates and found the monogamous graph bound to a matching at every source, rate, and dilution. R = 0 is the entire mechanism, and the finding stands rescoped as a property of over-driven coupling rather than of the dynamics as such.

4.2 Three phases. Observed, with the mechanism above.

Small theta, retention near one: strong pairs survive tens of foreign firings, the birth matching freezes into a dimer crystal, degree exactly 1.00 at every size, tangle 0.62 to 0.85 per edge, nothing beyond direct partners. The set-point, retention 0.61: pairs survive about two firings, a liquid of forming and dissolving half-strength edges, degree 1.0 to 1.2. Beyond, retention collapsing: the over-coupled regime, degree falling to 0.65, the matching turned over faster than it can thicken.

4.3 The set-point is the optimum. Observed at four-stream precision.

The degree is maximal at theta* and nowhere else in the sweep (Figure 3, lower panel). Pooled over four trajectory streams the set-point degree reads 1.12 at twelve qubits and 1.01 at sixteen, straddling one, with one stream dipping to 0.93. The excursion above the matching line is real, small, and quoted here at exactly the precision the data supports. The coupling that a set-point of half the pair maximum specifies is therefore vindicated twice over: it is the faithful reading of the update rule, and it is where the monogamous graph is densest. That the optimum still does not percolate is the subject of the next section, and the reason is not a missing ingredient.

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5. The pinning law

5.1 Budget saturation. Observed everywhere.

In every live regime the largest per-vertex tangle sum sits between 0.90 and 1.00 and never exceeds one, the tightest running confirmation of the monogamy budget this study has. The budget is spent in full, always. What varies is how it is spent, concentrated on one near-maximal partner or spread across a few half-strength ones.

5.2 The age model and the logarithm. Mean-field, four regimes consistent.

Give a new edge tangle s(theta) at birth and an effective per-step retention Rc from gate hits and channel together. It counts toward the degree while above the floor tau_f. Then

d = ln(s / tau_f) / ln(1 / Rc),

a ratio of logarithms. Source model, source rate, dilution strength, and coupling all enter inside a logarithm, and the budget clamps the numerator. This is the pin: not a conspiracy of parameters but a functional form that moves by decades of its arguments while every dial moves by factors. Inverting the measured degrees across four regimes gives effective retentions of 0.002 to 0.018, self-consistent with the model, and the campaign's full range of measured degrees, 0.6 to 1.2 across ten rounds, is what a logarithm looks like from outside (Figure 4).

5.3 One residual, booked open

The single-hit retention at the set-point, 0.61, implies a naive per-step retention near 0.37 for a two-endpoint edge, a factor of order twenty above the inverted effective value. Joint hits are not independent hits: re-pairing steals budget collectively, and the joint map is the one derivation this study names and does not supply. Open.

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6. The distance dichotomy

The pinning results say how many partners a vertex holds. The decay profile says where weight lives, and the answer splits along exactly the line the two theorems drew. Bin the pairwise weights of every trajectory by hop distance on the mutual-information graph and the monogamous weight vanishes beyond hop one: mean tangle 0.20 to 0.85 at direct partners, identically zero at hops two and three, in every regime, every stream, every size measured. Mutual information leaks. In the liquid it falls from 0.41 nats at hop one to 0.03 at hop two to 0.015 at hop three, a decade per hop, exponential (Figure 5). In the frozen phase nothing exists beyond hop one at all: the dimer crystal has no second neighbors on its own graph.

One consequence lands on the companion framework's axiom directly. A4 asks the surviving weight to fall off as a power law in graph distance, the behavior a manifold limit needs. At every coupling this study reached, the falloff of the polygamous weight is exponential and the monogamous weight has no falloff to speak of, only a wall. Power-law tails are a property of critical points, and Section 7 measures how far from criticality the set-point graph sits. The tension is booked rather than resolved: either the power-law clause holds only in a regime this dynamics does not reach, or it holds for a weight this study did not measure. Open.

The dichotomy is visible whole in a single trajectory. Figure 6 draws the actual graphs of one archived run at three couplings, re-derived bit-exactly from the stored seeds: the frozen crystal, eight heavy chords and nothing else; the liquid, a matching thickened by half-strength edges with a faint polygamous web behind it; the full gate, a sparse weak matching, the deletion mechanism at work. The picture is the theorems wearing the dynamics.

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7. The Erdős–Rényi correction

Degree one is the percolation threshold of a random graph, and an earlier round of this campaign read the set-point's degree 1.0 to 1.2 as criticality within reach. The comparison test corrects the reading. Erdős–Rényi Monte Carlo at matched size and matched mean degree gives giant components of 0.41 to 0.52. The measured graphs give 0.19 to 0.27, the fourteenth percentile of the null (Figure 7). The set-point graph is not a random graph near its transition. It is more fragmented than random at equal degree, because its degree is not Poisson-distributed: the budget concentrates edges as one per vertex, a matching with occasional excursions, and matchings have no giant.

The correction sharpens the study's conclusion instead of weakening it. What percolation requires is not a nudge of mean degree past one, which the set-point already delivers, but breaking the matching concentration itself, and the concentration is not a parameter. It is what sequential two-body firing does: one partner at a time, budget monopolized per firing, prior edges shed by the same gate that writes the new one. The failure mode was never fuel, rate, dilution, or coupling. It is arity.

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8. The elimination, and the one lever left

Ten rounds, each pre-registered, each ended at its stop rule, fifteen ledger entries, eight of the campaign's own written expectations falsified. The map they leave: purity is the fuel and only a source supplies it. Replacement sources cancel. Fixed-rate sources lose to volume. Volumetric sources give a live phase at a derived fixed point. Inside the live phase the budget saturates and the degree answers every dial with a logarithm. The weight that could carry geometry sits on a matching and does not spread. And the matching is what two-body gates write, at any strength, under any dilution, conditioned or not.

Everything pairwise has been turned. What has never been turned is the arity of the interaction itself. A three-body step spends its budget on a triangle rather than an edge, and a graph of overlapping triangles is connected at densities where a graph of disjoint edges is dust. The companion framework's own third-order machinery, built for a different question, measures precisely the quantity a three-body step would write: interaction information, the weight that exists on a triple and on no pair inside it. Whether such a step can percolate the monogamous graph while Theorem 2 holds it to bounded degree is a sharp, decidable question, and it is the single question this study hands forward. The elimination is the result. The lever is named. The next simulation decides.

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9. Methods

9.1 Protocol

Every round was pre-registered before its build: gate family, cells, seeds, atomic pass and kill criteria with numbers, and the author's written expectations, all committed to a dated log before the first line of probe code. Rounds end at their stop rule regardless of momentum. No retuning after a verdict. A negative result is a result. Surprises reproduce on fresh seed streams before interpretation. Supportive outcomes carry an extra guard, declared in advance wherever the author's written lean favored support: reproduction on a second stream plus the relevant null comparison before booking. Two rounds were falsified by their own instruments and rebuilt: a gate family that could not entangle its calibration state, caught one sweep late because a unit check printed instead of asserting, and a probe patch that failed silently in a shell error, caught by a filename that carried the old format. Both event chains are in the log. Both purged run sets are named below. The lesson, unit checks assert and halt, is part of the protocol now.

9.2 Simulators and scale

Exact density matrices to twelve qubits. Pure-state trajectories, an exact unraveling of per-qubit depolarizing into stochastic Paulis, to sixteen qubits at ten trajectories per cell, verified against the exact channel to 1.5e-3 at twenty thousand samples. Births follow a deterministic accumulator so system size is common across trajectories and ensemble marginals are unbiased averages of trajectory marginals. Edge floors: mutual information 0.05 nats, tangle 0.0025. Coupling calibration: theta* = 1.3524 solves I(one firing on a fresh pair) = ln 2 to 1e-6, hard-asserted.

9.3 Scripts, data, verification

Probes: oracle2_state_probe (channels, sources, the death diagnostic), round3_growth (replacement, fixed-rate, and volumetric genesis; patch_vol documents the volumetric extension), round5_trajectory (trajectory layer, ensemble averaging), round6_a3graph (per-trajectory graphs), round7_theta (the coupling sweep with calibration), round8_critical_dial (slow dilution and the hop-distance profile, reused for the frozen cells). Analysis and figures: campaign1b_analysis, oracle1_sanity, oracle2_analysis, m1_support, a_o1_figures and a_o1_figures2, the latter re-deriving Figure 6's trajectories from archived seeds and asserting exact equality against archived per-trajectory rows. Every verdict number in this study was recomputed from archived npz output by an independent code path (the o1b, o3, and r2 through r5 cold-verification files and coldpass_tail). Headline cells run three to four seed streams (stat_hardening). Seed registry: 9101 to 9110 growth, 9201 to 9208 volumetric, 9301 and 9401 trajectories, 9701 to 9713 and 9801 to 9803 the coupling sweep and its hardening, 9901 to 9912 slow dilution, 10001 to 10011 the frozen cells. Purged as invalid and rerun: the pre-patch volumetric pair 9205 and 9206, and the first-family sweep 9701 to 9703. One archival gap is declared rather than hidden: the hop-distance profiles of rounds eight and nine were not written to npz, only to the analysis text files. The runs are deterministic given their seeds and the profiles reproduce to four decimals, but a reader wanting the raw per-pair weights must rerun the archived scripts.

9.4 The anomaly ledger

Fifteen entries, all reproduced, none silenced, each with observation, written expectation, diagnosis, and promotion. In one line each: no budget for mutual information; the broadcast blindness; unital death and the reset-depolarizing identity; the concealed source in set-point update rules; replacement's rate cancellation; fixed-rate loss to volume; the volumetric fixed point and the cap artifact that first hid it; source-rate saturation of graph density and the trajectory-ensemble factor; the matching bound under full gates, later rescoped; the over-driven coupling flag; the set-point optimum; dilution independence and hop-one confinement; the frozen dimer phase and the pinning synthesis; the Erdős–Rényi correction, superseding the campaign's own criticality reading; and the four-stream softening of the set-point excursion. Eight of the fifteen falsified the author's written expectations. The ledger is the study's spine, not its appendix waste.

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10. Neighbors, and what is claimed

Cao, Carroll, and Michalakis (2017) build space from the same mutual-information weight and restrict by hand to redundancy-constrained states. Quantum graphity (Konopka, Markopoulou, Smolin) stabilizes low degree with an energy penalty. Sung-Sik Lee (2018) remarks qualitatively that monogamy should tame entanglement-based geometry. The measurement-induced-transition literature (Skinner, Ruhman, Nahum; Li, Chen, Fisher) established that trajectory ensembles carry entanglement that ensemble states do not, and owns the Clifford-scale toolbox this study's extension would use.

Against those neighbors, the claims grade as follows. The mathematics of Theorems 1 and 2 is corollary-grade: broadcast states, Araki-Lieb, Christandl-Winter, and Coffman-Kundu-Wootters in Osborne-Verstraete form are all standard, and the theorems' value is placement, not depth. Graded candidate-novel per this campaign's literature scans, and explicitly uncertified pending adversarial expert review: the diagnosis that one polygamy theorem explains why three independent programs impose regularity by hand. The derivation chain from unitality through volumetric genesis to the fixed point r* = lambda / (lambda + b) as the existence condition of the live phase. The exact retention curve R(theta) with its three phases and the identification of the half-maximum set-point as the degree optimum. And the pinning result itself, the matching confinement of monogamous weight under sequential pairwise dynamics across every dial. The trajectory-ensemble gap is the transition literature's insight and is claimed only as transposed. Certification requires readers with incentives to break these results. The study is written to make that as easy as possible.

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11. Open problems

Seven, in decreasing sharpness. The multipartite question, handed forward as this study's successor: can a low-density three-body step percolate the monogamous graph while the budget theorem holds its degree bounded, and does the interaction-information weight it writes decay with distance as geometry needs. The joint-hit map: single-hit retention 0.61 against effective per-step retention 0.018, a factor twenty carried by correlated re-pairing, underived. The power-law regime: whether any coupling and any weight in this dynamics class produces the power-law falloff a manifold limit requires, or whether exponential decay and the hop-one wall are the class's last word. The set-point coincidence: whether the degree optimum at half-maximum coupling follows from the retention-injection trade-off in closed form, or is a numerical accident of this gate family. Scale: sixteen qubits and ten trajectories bound every claim here. The Clifford-sector extension with controlled non-Clifford density is the named tool. The r* coefficient: the operator-weight bracket 16p/3 to 8p contains the fitted 6.0 to 6.2, and closing the bracket means deriving the deficit's weight dynamics. And the phase diagram: the frozen, liquid, and annihilation boundaries were located along two axes. The full (theta, eta) plane is unmapped.

The study set out to derive one axiom and ends holding half of it as a theorem, the other half as a named obstruction with a single untested lever. That is the honest shape of the result, and the lever is where this line of work goes next.

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Figure Appendix

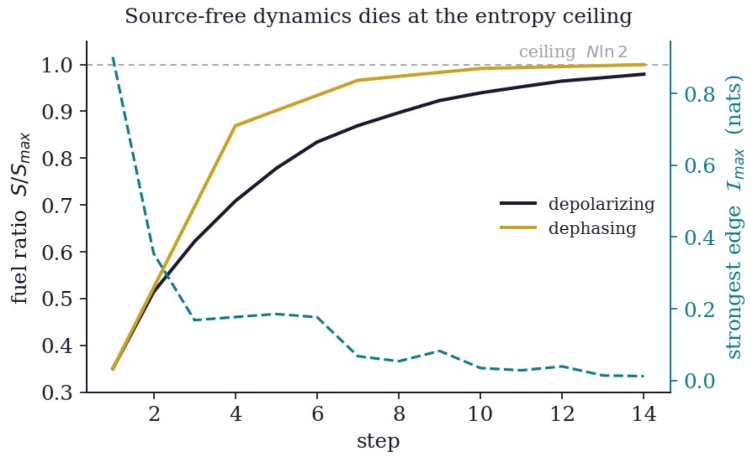

Figure 1. Death under dilution

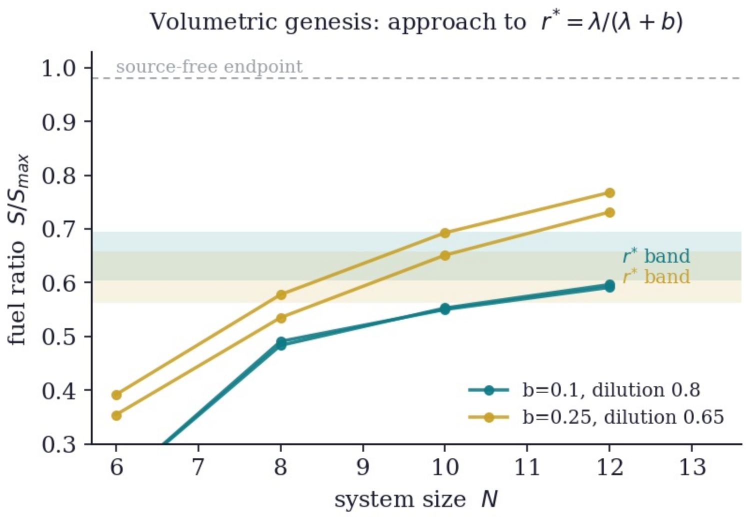

Figure 2. Live phase fixed point

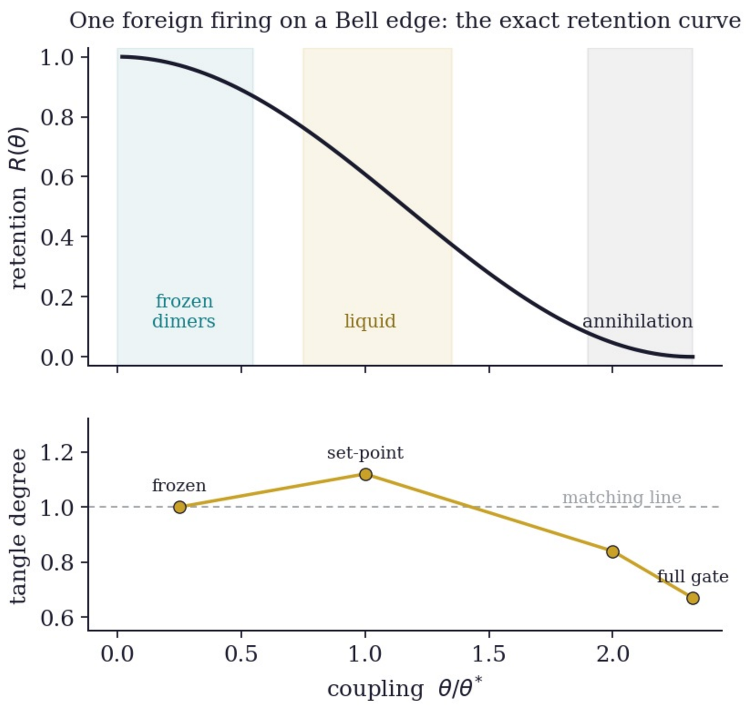

Figure 3. Retention and set-point

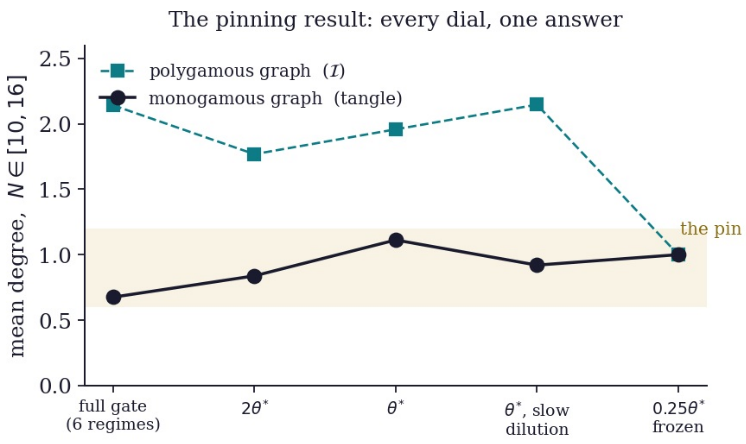

Figure 4. Pinning law

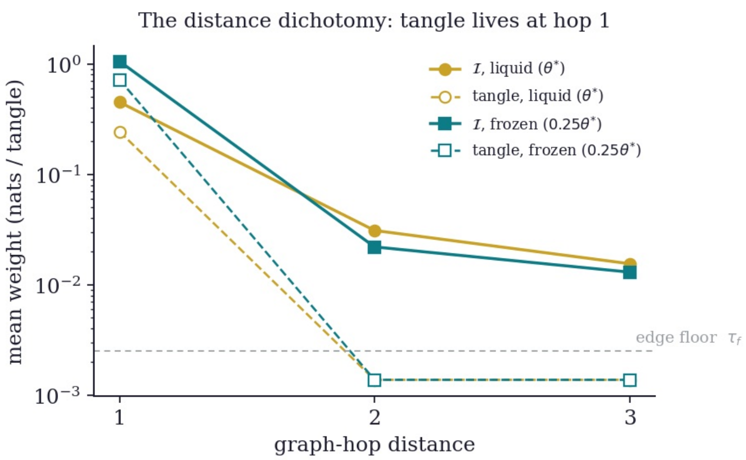

Figure 5. Hop decay

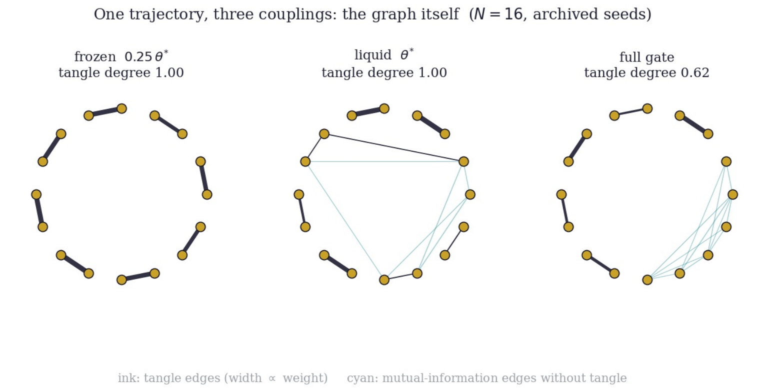

Figure 6. Graph comparison

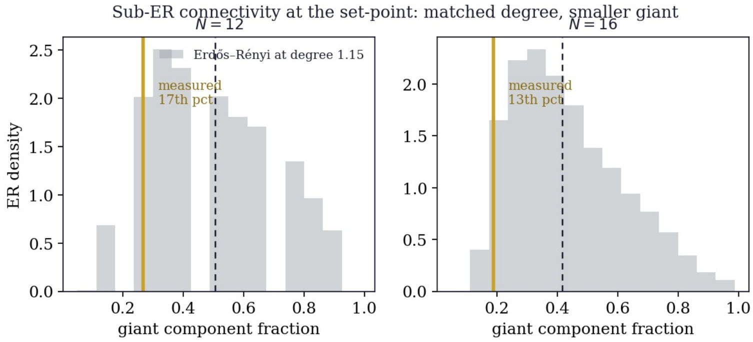

Figure 7. Erdos-Renyi correction